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JET EVOLUTION, FLUX RATIOS AND LIGHT-TRAVEL TIME EFFECTS

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ABSTRACT

Studies of the knotty jets in both quasars and microquasars frequently make use of the ratio of the intensities of corresponding knots on opposite sides of the nucleus in order to infer the product of the intrinsic jet speed ($\beta_{\rm jet}$) and the cosine of the angle the jet-axis makes with the line-of-sight ($\cos\theta$), via the formalism $I_{\rm a}/I_{\rm r} = ((1+\beta_{\rm jet}\cos\theta)/(1-\beta_{\rm jet}\cos\theta))^{3+\alpha}$, where α relates the intensity I_ν as a function of frequency ν as $I_\nu \propto \nu^{-\alpha}$. In the cases where $\beta_{\rm jet}\cos\theta$ is determined independently, it is found that the intensity ratio of a given pair of jet to counter-jet knots is over-predicted by the above formalism compared with the intensity ratio actually measured from radio images. As an example in the case of the microquasar Cygnus X-3 the original formalism predicts an intensity ratio of ~ 185, whereas the observed intensity ratio at one single epoch is ~ 3. Mirabel & Rodríguez (1999) have presented a refined approach to the original formalism which involves measuring the intensity ratio of knots when they are at equal angular separations from the nucleus. This method is however only applicable where there is sufficient time-sampling (with sufficient physical resolution) of the fading of the jet-knots so that interpolation of their intensities at equal distances from the nucleus is possible. This method can therefore be difficult to apply to microquasars and is impossible to apply to quasars. We demonstrate that inclusion of two indisputable physical effects: (i) the light-travel time between the knots and (ii) the simple evolution of the knots themselves (e.g. via adiabatic expansion) reconciles this over-prediction (in the case of Cygnus X-3 quoted above, an intensity ratio of ~ 3 is predicted) and renders the original formalism obsolete.

Subject headings: methods: analytical — radiation mechanisms: non-thermal — relativity — ISM: jets and outflows

1. INTRODUCTION

Relativistic jets are observed in both quasars and microquasars, and are often seen to consist of a series of discrete knots moving outwards from a central nucleus, believed to correspond to the compact object powering the outflow. Measurements of the proper motions of these knots are often used to constrain properties such as jet speeds and inclination angles, and source distance (e.g. Mirabel & Rodríguez 1999; Hjellming & Rupen 1995). The ratio of the intensities of approaching and receding knots (if there is sufficient spatial resolution that these can be accurately identified) have been used (e.g. Saripalli et al. 1997) to constrain their Lorentz factors, via

$$\frac{S_{\rm app}}{S_{\rm rec}} = \left(\frac{\mu_{\rm app}}{\mu_{\rm rec}}\right)^{k+\alpha} = \left(\frac{1+\beta\cos\theta}{1-\beta\cos\theta}\right)^{k+\alpha}, \qquad (1)$$

where $\beta = v/c$ is the jet speed, θ is the inclination angle of the jet axis to the line of sight, α is the spectral index of the emission (defined by $S_\nu \propto \nu^{-\alpha}$, where S_ν is the flux density at frequency ν), $S_{\rm app}$ and $S_{\rm rec}$ are the flux densities of a corresponding pair of approaching and receding knots, $\mu_{\rm app}$ and $\mu_{\rm rec}$ are their proper motions, and $k = 3$ for a jet composed of discrete ejecta.

The luminosities $L(t)$ of the knots change with time t, as the knots expand and the magnetic field, and hence the synchrotron emissivity, decreases. Thus the true flux ratio is

$$\frac{S_{\rm app}}{S_{\rm rec}} = \left(\frac{1+\beta\cos\theta}{1-\beta\cos\theta}\right)^{k+\alpha} \frac{L_{\rm app}(t_{\rm app})}{L_{\rm rec}(t_{\rm rec})}, \qquad (2)$$

where $t_{\rm app}$ and $t_{\rm rec}$ are the times at which light leaves the approaching and receding knots respectively in order to arrive at the telescope at the same time. Unless the jet axis is perpendicular to the line of sight however, the light-travel time between approaching and receding knots will mean that we see the receding jet as it was at an earlier time, when it was more compact and hence intrinsically brighter (but also dimmed in the observer's frame by its recessional motion, taken into account by the original formalism), compared with the approaching jet seen at the same telescope time. To account for this effect, Mirabel & Rodríguez (1999) proposed that the flux densities used to calculate the ratio should be measured at equal angular separations from the nucleus. This cannot always be implemented in practice however, since this will require interpolation unless good temporal coverage of the jets is available, or unless the jet is a continuous flow, in which case the motion of individual knots cannot be tracked in any case. At early times it may also be difficult to separate the emission from moving jet knots and a fading core if there is insufficient spatial resolution. Moreover, as a result of opacity or the presence of a broken power law, interpolation of the spectrum may not be straightforward if in the observer's frame we sample different parts of the spectrum at any given frequency.

In this *Letter*, we present a method of using the flux ratios from a single image of a source to constrain the jet speeds without resorting to interpolation via the Mirabel & Rodríguez method.

2. FLUX RATIOS

2.1. *Simple Scalings*

A synchrotron-emitting plasmon where the particles undergo adiabatic expansion will have a power law decay in intensity, $L(t) \propto t^{-\zeta}$, in which case equation 2 becomes

$$\frac{S_{\rm app}}{S_{\rm rec}} = \left(\frac{1+\beta\cos\theta}{1-\beta\cos\theta}\right)^{k+\alpha} \left(\frac{t_{\rm app}}{t_{\rm rec}}\right)^{-\zeta}, \quad (3)$$

and this scaling will apply to any process which gives a power law decay in intensity. We consider symmetric approaching and receding jets, in which case after ejection at $t = 0$, the epochs at which photons leave corresponding points of the front and back plasmons, $t_{\rm app}$ and $t_{\rm rec}$ respectively, are related by

$$\frac{t_{\rm app}}{t_{\rm rec}} = \frac{1+\beta\cos\theta}{1-\beta\cos\theta}. \quad (4)$$

So in this simple case equation 3 becomes

$$\frac{S_{\rm app}}{S_{\rm rec}} = \left(\frac{1+\beta\cos\theta}{1-\beta\cos\theta}\right)^{k+\alpha-\zeta}. \quad (5)$$

While this ratio is applicable to any process that gives a power law decay, in the adiabatically expanding synchrotron case the parameters α and ζ are not independent so that the determination of the flux ratios by equation 5 is actually not introducing an extra parameter.

2.2. *Optically thin synchrotron emission and adiabatic expansion*

The total synchrotron emissivity from a single, optically thin jet knot scales as (e.g. Longair 1994)

$$J(\nu) \propto B^{3/2} N(\gamma)\gamma^2 \nu^{-1/2}, \quad (6)$$

where B is the magnetic field strength, γ is the Lorentz factor of an individual electron assumed to be radiating at a single frequency

$$\nu = \left(\frac{\gamma^2 eB}{2\pi m_{\rm e}}\right), \quad (7)$$

and $N(\gamma)$ is the total number of electrons with energies in the range $(\gamma, \gamma + d\gamma)$ in the plasmon, given by

$$N(\gamma, t_0) = A\gamma^{-p}, \quad (8)$$

where p is the electron index, t_0 is some arbitrary reference time, and A is the normalisation constant. As the plasmon expands, nonrelativistically, from a radius $R_0(t_0)$ to $R(t)$ the electron energy scales as

$$\gamma = \frac{R_0}{R}\gamma_0, \quad (9)$$

if synchrotron losses are negligible. The spectrum then evolves according to

$$N(\gamma, t) = \left(\frac{R}{R_0}\right) N\left(\frac{R}{R_0}\gamma_0, t_0\right). \quad (10)$$

Putting all of the above together we find that the synchrotron emissivity of an expanding plasmon is given by

$$J(\nu) \propto \nu^{(1-p)/2} B^{(1+p)/2} R^{1-p}. \quad (11)$$

As the plasmon expands the magnetic field strength will decrease and, in the case of a tangled field, we have $B \propto R^{-1}$, so the plasmon emissivity has a simple dependence on frequency and size given by

$$J(\nu) \propto \nu^{(1-p)/2} R^{(1-3p)/2}. \quad (12)$$

The ratio of flux densities as seen by the observer is then

$$\frac{S_{\rm app}}{S_{\rm rec}} = \left(\frac{R(t_{\rm app})}{R(t_{\rm rec})}\right)^{(1-3p)/2} \left(\frac{1+\beta\cos\theta}{1-\beta\cos\theta}\right)^{k+(p-1)/2}. \quad (13)$$

Although we could take the expansion of the plasmon to be of the form $R \propto t^{\eta}$, it is particularly instructive to look at the case of linear expansion, $\eta = 1$, for which equation 13 becomes

$$\frac{S_{\rm app}}{S_{\rm rec}} = \left(\frac{1+\beta\cos\theta}{1-\beta\cos\theta}\right)^{k-p}. \quad (14)$$

This is the flux ratio observed at a given instant by the telescope as opposed to the interpolated flux at equal angular separations. As a simple generic case, emission from a jet composed of discrete ejecta ($k = 3$) from a spectrum of electron index $p = 2$ will give an exponent of unity for equation 14. By way of contrast, obtaining an interpolated estimate at equal angular separations will, in this case, give an exponent of $k + \alpha = 3.5$. The flux ratios to be measured in the two cases would, however, differ, being measured in a single image in the former case and at equal angular separations in the latter. A comparison of both methods in any given source would of course be a useful means of inferring a possible asymmetry in the approaching and receding jets (Atoyan & Aharonian 1997).

2.3. *Synchrotron Self-Absorption and Spectral Breaks*

When the particle spectrum contains a break or a turnover we must adapt the above discussion. For example, in Cygnus X-3 (Miller-Jones et al. (2004)) we observe two discrete knots, one on each side of the central nucleus, with evidence for a turnover due to synchrotron self-absorption. In this case the spectrum will have the form $J_\nu \propto \nu^{5/2}$ below the turnover frequency ν_0 when the knot radius is R_0 while above this frequency the spectrum is optically thin, $J_\nu \propto \nu^{-(p-1)/2}$. As the knot expands to a radius R its emission will then take the self-absorbed form

$$J(\nu, R) = J_{\rm max}(R) \left(\frac{\nu}{\nu_c(R)}\right)^{5/2}, \quad \nu \leq \nu_c(R), \quad (15)$$

while in the optically thin regime the intensity scales like

$$J(\nu, R) = J_{\rm max}(R) \left(\frac{\nu}{\nu_c(R)}\right)^{-(p-1)/2}, \quad \nu \geq \nu_c(R). \quad (16)$$

The critical frequency beyond which the emissivity becomes optically thin is determined by

$$\nu_c(R) = \nu_0 \left(\frac{R}{R_0}\right)^{-(3p+4)/(p+4)}, \quad (17)$$

and the emission at that frequency, which is the peak of the knot spectrum, becomes

$$J_{\rm max}(R) = J_0 \left(\frac{R}{R_0}\right)^{-5p/(p+4)}. \quad (18)$$

Turning now to the flux ratios observed at a given instant and frequency it is clear that at sufficiently early and late times we will have two extremes. In the former case,

when the observed emission from each knot is optically thick we will see a $J_\nu \propto R^{5/2}\nu^{5/2}$ spectrum from each and the flux ratio exponent is $k = 3$ for discrete ejecta. However, it may be difficult to observe actual knots in this regime without mixing in possible nuclear emission. At observed frequency ν the emission will become optically thin from the approaching knot when its radius is R_1 which is determined by

$$\nu = (1+\beta\cos\theta)\nu_0 \left(\frac{R_1}{R_0}\right)^{-(3p+4)/(p+4)}, \qquad (19)$$

while the emission from the receding knot will remain optically thick, at this frequency, until the approaching knot has a radius of R_2 which can be easily shown to be

$$R_2 = \left(\frac{1+\beta\cos\theta}{1-\beta\cos\theta}\right)^{2p/(3p+4)} R_1. \qquad (20)$$

Therefore, when the approaching knot has a radius R satisfying $R_1 \leq R \leq R_2$ the observed emission at frequency ν will be a mix of Doppler boosted, optically thin emission from the forward knot and optically thick emission from the receding component. The flux ratio in this regime is now dependent on time, i.e. knot radius, and is given by

$$\frac{S_{\rm app}}{S_{\rm rec}} = \left(\frac{1+\beta\cos\theta}{1-\beta\cos\theta}\right)^{k} \left(\frac{R}{R_1}\right)^{-(3p+4)/2}. \qquad (21)$$

At $R = R_2$ all of the emission becomes optically thin at this frequency and the flux ratios predicted by equations 14 and 21 are equal. Therefore, the flux ratio exponent drops from a value of k to $k-p$ as the front and then the receding knot become optically thin. During this time the spectrum at frequency ν also evolves from $\nu^{5/2}$ to $\nu^{-\alpha}$ and the forward knot expands by a factor R_2/R_1, where both radii are frequency dependent. The time taken for this expansion is determined by the expansion velocity V of the knot.

In reality however, it is unlikely that the turnover in the spectrum would occur at one single frequency. There would be a finite turnover region in which the spectrum evolved from a $\nu^{5/2}$ power-law to $\nu^{-(p-1)/2}$. Depending on the width of the turnover region and the value of $\beta\cos\theta$, the receding knot could be in the turnover region of the spectrum by the time the approaching knot had become optically thin. In this case, the above results would not be strictly applicable.

Nonetheless, these general points apply to any source of opacity which changes the spectral shape or indeed to any broken power law that might be attributable to the acceleration mechanism. It presents the possibility that the evolution of the spectrum from flares in microquasars may well be influenced by the light-travel time differences between approaching and receding knots, as outlined in Miller-Jones et al. (2004).

2.4. *Caveats*

Care should be taken if the expansion mode of the plasmons changes prior to the observation from which the flux ratio is derived. In such a case, for example a transition from slowed to free expansion (Hjellming & Johnston 1988), the time decay of the flux density would change (steepen with time in this case). In order to use flux ratios to constrain the value of $\beta\cos\theta$, the flux densities of the approaching and receding knots would then have to be measured when the knots were both in the same expansion regime. Unless the transition radius were known, this would require actually measuring (as opposed to interpolating) the flux densities at equal angular separation from the core. We note that if there is significant deceleration of the expanding plasmons due to interaction with surrounding material, as mentioned by Hjellming & Han (1995), then $(R/R_0) \propto (t/t_0)^\eta$, where $\eta < 1$, and equation 14 then requires modification. We also draw attention to Fender (2003), which presents caveats to be considered when using proper motions to place limits on the bulk Lorentz factors of jets; any Lorentz factors thus derived are strictly only *lower limits*.

3. COMPARISON WITH OBSERVATIONS

The VLBA observations of Cygnus X-3 presented by Miller-Jones et al. (2004) show a jet which at 5 GHz and 15 GHz appears to be composed of two separating discrete knots, which were interpreted as approaching and receding plasmons. A precession modelling analysis yielded a value $\beta\cos\theta = 0.62 \pm 0.11$, and the spectral index of the emission was found to be $\alpha = 0.60 \pm 0.05$. Assuming linear expansion of the jet knots, we would thus predict a flux density ratio of 3.2 ± 1.0. For the last two epochs (2001 September 20 and 21), the measured flux ratios are given in Table 1. While not matching the theoretical prediction perfectly, they are now of the correct order, in contrast with the predictions of the original formalism, which is wrong by two orders of magnitude. There are various possible explanations for the slight discrepancy. Most importantly, the measurement of the flux densities themselves was often difficult. It is also possible that the plasmon expansion was not exactly linear with time, which would alter the exponent in equation 14 and change the predicted flux density ratio. Moreover, the measured spectral index α was for the integrated spectrum; the values of α and p could in principle differ for the individual jet knots. The quality of the data makes it difficult to interpolate back to the flux densities at equal angular separations in this case, but our best attempts gave flux ratios between 1.58 and 10.63. In such cases, our direct measurement method gives a much more accurate determination of the expected flux ratio, for more meaningful comparison with the jet speeds and inclination angle found by different methods. We note that if the spectral index of the jet material is known, our method requires only a single image to determine the value of $\beta\cos\theta$, whereas the interpolation method requires at least two images taken at different times. This frees it from the uncertainty inherent in comparing VLBI images, particularly if the imaging is difficult, as was the case in these observations (Miller-Jones et al. 2004). For a single image, the ratio of two flux densities is set, whereas when comparing different images, in order to be able to interpolate accurately, one has to be confident that one has recovered the same fraction of the true flux density in both images in order to be able to take an accurate flux density ratio.

This theory could also be applied to the observations of GRS 1915+105 detailed by Mirabel & Rodríguez (1994). They observed discrete radio ejecta moving outward from the nucleus over a period of ~ 1 month. Again, we take

the flux density ratio of their observed knots once they had clearly separated from one another and from the nucleus, and we only compare corresponding pairs of ejecta. From their derived value of $\beta \cos\theta = 0.323 \pm 0.016$ and their quoted spectral index of $\alpha = 0.84 \pm 0.03$, we predict a flux density ratio of 1.24 ± 0.05. For the later epochs (1994 April 16, 23 and 30), the measured flux density ratios are 2.33, 2.63 and 1.80 respectively. Again, this is slightly greater than we predict, but is of the right order. Underpredicting the flux density ratio implies the exponent should be larger in equation 14, which requires $\eta > 1$, i.e. the expansion scales slightly more rapidly with time than $R \propto t$.

4. CONCLUSIONS

We have considered the evolution of synchrotron bubbles (plasmons) in oppositely-directed microquasar jets. We have found that our new formalism can explain the observed flux density ratios in microquasar jets in systems in which the synchrotron bubble model is applicable, such as Cygnus X-3. In contrast, the original formalism considerably overpredicts the observed flux density ratio in observations of this system. In the case of free (linear) expansion, $(R/R_0) \propto (t/t_0)$, we found that the flux ratios of the approaching and receding plasmons are given by $S_{\rm app}/S_{\rm rec} = ((1 + \beta\cos\theta)/(1 - \beta\cos\theta))^{k-p}$.

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REFERENCES

Atoyan, A.M., & Aharonian, F.A. 1997, ApJ (Letters), 490, L149
Fender, R.P. 2003, MNRAS, 340, 1353
Hjellming, R.M., & Han, X. 1995, in Cambridge Astrophysics Ser. 26, X-ray binaries, ed. Lewin, W.H.G., van Paradijs, J., & van den Heuvel, E.P.J. (Cambridge: CUP)
Hjellming, R.M., & Johnston, K.J. 1988, ApJ, 328, 600
Hjellming, R.M., & Rupen, M.P. 1995, Nature, 375, 464
Longair, M.S. 1994, High energy Astrophysics, Vol. 2: Stars, the Galaxy and the interstellar medium(2nd ed.; Cambridge: CUP)
Miller-Jones, J.C.A., Blundell, K.M., Rupen, M.P., Mioduszewski, A.J., Duffy, P. & Beasley, A.J. 2004, ApJ, in press (astro-ph/0311277)
Mirabel, I.F. & Rodríguez, L.F. 1994, Nature, 371, 46
Mirabel, I.F. & Rodríguez, L.F. 1999, ARA&A, 37, 409
Saripalli, L., Patnaik, A.R., Porcas, R.W., & Graham, D.A. 1997, A&A, 328, 78

TABLE 1
CYGNUS X-3 VLBA FLUX DENSITY RATIOS MEASURED FROM OBSERVATIONS OF 2001 SEPTEMBER OUTBURST OF CYGNUS X-3

Date (UT)	Observing Frequency (GHz)	Flux density ratio (South/North)
September 20	5	1.43 ± 0.05
September 21	5	2.39 ± 0.10
September 20	15	1.14 ± 0.19
September 21	15	3.09 ± 0.14